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Research Article

# Prediction Of E-Payment Channels with Hidden Markov Model

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## Abstract:

Hidden Markov models (HMM) are a class of stochastic modelling that describe the evolution of observable events that depends on internal factors (hidden states) which are not directly observable. This probabilistic model has use in a wide range of disciplines, including engineering, finance, and medicine. In this study, HMM is employed in e-payment transactions to see the trends of these e-payment systems and to determine what would be the future trend of the e-payment channels. Two e-payment channels were considered: Automated teller machine (ATM) and point of sale (PoS) machines which are the observable state while the hidden states are high and a low volume of transactions. The emission matrix was determined based on the observable state count and the transition probability count. The filtering method was employed in the HMM to estimate the parameters of the model. Empirical analysis revealed that the volume of transactions for PoS surpass that of ATM based on the data used in this study. The steady-state probability also shows that in the future the volume of transactions for PoS with the probability of 0.875 (87.5%) would surpass that of ATM with the probability of 0.125 (12.5%).

**Keywords: electronic payment, Transaction; behavioural modelling; e-commerce; hidden Markov model; filtering.**

## 1. Introduction

The term electronic payment system, often known as e-payment, refers to a type of financial commitment in which the buyer and the seller are assisted by use of technological means,
Kabir, et al. (2015). It involves the provision of fund transfer by means of phones, computers, the internet, automated teller machines (ATMs) and smartcards. Commercial payments through e-commerce now take the form of electronic monetary exchanges. Currently, the majority of businesses, organizations, and governmental bodies use electronic commerce to boost productivity when trading goods and services in sectors like credit cards, communications, healthcare insurance, auto insurance, online auctions, etc. Abdallah et al. (2016). The effectiveness of an electronic payment system is determined by how successfully it handles the difficulties presented by various online payment options.

In Nigeria, cash has long been the most common method of payment, which has advantages and disadvantages. The advantages of swift conversion to other forms of value without the need for a financial institution's middleman outweigh the disadvantages of anonymity and untraceability in immoral activities.

Due to a deluge of complaints about corruption in the Federal Civil Service, the administration decided to establish electronic payments. According to the Federal Government's Treasury Circular issued October 22, 2008, as of January 1, 2009, all payments made with Federal Government monies have to be made online. The policy's lack of preparation, inefficiencies, and payment delays for goods and services have garnered a lot of criticism.

While e-payment might be a more convenient option, concerns have been raised about the stability of its value. Mancini-Griffoli and Adrian (2019). Therefore, accurately predicting the price of digital currency can avoid risks for investors and earn huge assets. The banking industry may be able to enhance and increase its investments in the channel that yields more profits by using the dependable forecasts that modeling e-payment transactions provides.

As a result, this research will use the ATM and PoS e-payment channels available in Nigeria to understand their hidden conditions and forecast their future trends using a hidden Markov model.

## 2. Related Literature

Many prediction approaches from the simplest ones, such as the time-series analysis, through the more complex models such as artificial neural networks and machine learning algorithms, Bayesian networks and hidden Markov models have been used in analyzing E-payment channels in numerous research works.

Okifo and Igbunu (2015) looked at the barriers Nigerians faced when utilizing electronic payment systems and suggested solutions. Asaolu et al. (2011) investigated the difficulties the Nigerian Federal Government had in putting in place an electronic payment system and attempted to provide solutions.

Time-series are used in the clustering methods of Abbasimehr and Shabani (2021), and time-series forecasting techniques are then

applied to anticipate the behavior of each segment.
Using machine learning techniques including decision trees, cluster analysis, and the Naive Bayes algorithm, the authors of Jing et al. (2019) examined client traits and attributes with previous purchase records. By selecting models with high promotion degrees using a promotion graph model, they were able to further investigate the important variables impacting the purchasing behavior of potential clients.
Martinez, et al. (2020) looked into the framework for machine learning as well. In order to improve the prediction of whether the customer would make a purchase in the following month, a number of customer features that defined the customer at a specific month were computed using an already-existing customer transaction database. From these features, machine learning algorithms, including logistic Lasso regression, the extreme learning machine, and gradient trees, were applied.
Li et al. (2015) introduced a technique that utilizes a vast amount of user behavior log data to forecast the purchasing behavior of online to offline items for the following day. This study examined tree ensemble models (random forests and gradient boosting decision trees), logistic regression, and the Naive Bayesian classifier.
In order to determine the important drivers of online purchase intention, Dakduk et al. (2017) built on a study that combined the theory of planned behavior, the theory of reasoned action, and the technological acceptance model using a Bayesian method. In an online ordering application, Boyer and Hult (2005) presented behavioral scoring algorithms to predict customers' future purchases.
Hernandez et al. (2010) looked at the purchasing behaviors of two different customer types: seasoned internet shoppers and prospective shoppers who are considering making their first purchase online. New models in the field of behavioral modeling in e-commerce are becoming more and more popular as a result of the utilization of the Viterbi algorithm, Forney (1973, 2005) and hidden Markov models (HMM) Peentland and Liu (1999) to exploit Markov chains.
Xiao and Dong (2015) research on online to offline e-commerce presents a unique reputation management system based on a probabilistic hidden semi-Markov model. application.
In Srivastava et al. (2008), HMM was also utilized to detect credit card fraud. The HMM was trained using the cardholder's "normal" behavior with the aim of identifying suspicious actions indicated by low likelihood provided by the HMM. Mamata et al. (2003) proposed a web usage data mining model that took into account the activities of e-customers within a single site and used a discrete-time semi-Markov process to explain their behavior. The transition probability matrix and holding time mass functions were computed from the site navigation data. It was suggested by Danaa et al. (2021) to identify electronic banking fraud on highly imbalanced data using hidden markov models. HMM was utilized to model and forecast the detection of credit fraud (Siva Parvathi et al. (2012) Nkemnole and Akinsete, 2022).
Wang, Wu, and Yi (2018) employed the k-means algorithm with HMMs to detect fraud in online banking transactions. In their proposed approach, the observation symbols are a variable that stores the number of transactions within a window of time before and after each transaction, as well as the quantifiable values. If there is a sufficient chance for the trained HMM to reject an incoming transaction, it is considered fraudulent. They demonstrate the feasibility of their proposed method using simulated trials utilizing real-world bank transaction data. When there are sufficient prior transactions, their model works well for low, medium, and large user groups. An efficient prior estimation of the number of clusters is considered as a major difficulty in their proposed methodology.
In order to anticipate shop profits, Jandera and Skovranek (2022) developed the Customer Behavior Hidden Markov Model (CBHMM) to predict customer behavior in e-commerce. The model is composed of three sub-models: Loyalty, Vendor, and Psychology. These sub-models return probabilities that are utilized in the hidden Markov model's transition matrix to determine one of three decision-states: "Order completed," "Order uncompleted," or "No order." The predicted store income was assessed after the Viterbi algorithm analyzed the model outputs to determine whether the order had been fulfilled correctly. The GA model, which is the Google Analytics tracking system mechanism, served as the baseline prediction and was compared to the suggested CBHMM. Both the GA model and the predicted income calculated using CBHMM followed the real income data obtained from the store for the year 2021. Based on the comparison criteria their proposed CBHMM outperformed the GA model in terms of the R-squared criterion, giving a 5% better fit.
The recent Central bank policy on cashless policy deems it necessary to study the hidden trends in these payment channels, in order to ascertain which of these channels is widely popularized as regard usage in Nigeria.
Thus, in this research work, the hidden Markov model would be used to predict the trend of the e-payment (ATM and PoS) transaction channels in Nigeria in volume.

## 3. Methodology

### 3.1 Hidden Markov Chains

A hidden Markov model $\{X_t : t \in \mathbb{N}\}$ is a particular kind of dependent mixture with $X^{(t)}$ and $C^{(t)}$ representing the histories from time 1 to time *t*, one can summarize the simplest model of this kind by:

$$P\left(C_t | C^{(t-1)}\right) = P(C_t | C_{t-1}),\ t = 2,3,\ldots \qquad (1)$$

$$P\left(X_t | X^{(t-1)}, C^{(t)}\right) = P(X_t | C_t),\ t \in \mathbb{N} \qquad (2)$$

The model consists of two parts: firstly, an unobserved ‘parameter process’ $\{C_t : t = 1,2,\ldots\}$ satisfying the Markov property; and secondly, the ‘state-dependent process’ $\{X_t : t = 1,2,\ldots\}$, in which the distribution of $X_t$ depends only on the current state $C_t$ and not on previous states or observations. This structure is represented by the directed graph in Figure 1 below.

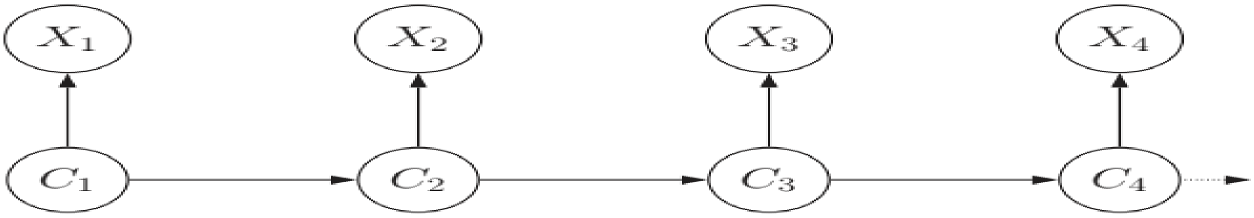


**Figure1: Directed graph of basic HMM (Source: *Zucchini, MacDonald & Langrock (2016)*)**

### 3.1.1 HMM Parameters

The Hidden Markov Model parameters are given by

$$\lambda = (N, M, A, B, \pi)$$

- N: The number of states. The number of hidden states is not always easy to determine. By considering the physical characteristics of the model, this can be set individually. This set is shown as $S = (s_1, \ldots, s_N)$ and at time $t+1$ the state is at $X_{t+1}$
- M: The number of distinct observations in each state. This is an output format of the physical modeling that it has done. The symbols are denoted as a set:

$$r_0^T = \{r_1, \ldots, r_M\}$$

- Transition Matrix: The probability of transition from state *i* to *j*. Transition probabilities are shown in matrix, see Equation 2.2:

$$A = \{a_{ij}\} = \begin{pmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{pmatrix} \qquad (3)$$

Each transition probability is calculated with the probabilities of getting from one state to another as seen in Equation (4)

$$a_{ij} = P\lfloor X_{t+1} = s_j \mid X_t = s_i \rfloor \quad ,1 \le i, j \le N \qquad (4)$$

For a special case that each step is accessible from other steps the following conditions should be qualified.

$$a_{ij} \ge 0 \qquad (5)$$

$$\sum_{j=1}^{N} a_{ij} = 1 \qquad (6)$$

If the state is unobtainable, then it is set to zero, as seen in equation (7).

$$a_{ij} = 0 \qquad (7)$$

- The emission probability is the observation symbol at state j, shown in equation (8).

$$B = (b_j(i)) \text{, where } (b_j(i)) = P\lfloor Y_t = r_k \mid X_{t+1} = S_j \rfloor, \qquad 1 \le j \le N, \qquad 1 \le i \le M \qquad (8)$$

- Initial State probability: The probability at state i in time zero $t=0$, gives Equation (9)

$$\pi_i = P\lfloor X_0 = S_i \rfloor, 1 \le i \le N \qquad (9)$$

By setting the N, M, A, B and π, the HMM can explain the observed sequence.

### 3.1.1 Formulation of Hidden Markov Model for payment transaction channels

In formulating the HMM model, the hidden states are defined as the category of the monthly volume of e-payment transaction channels for both ATM and POS and the observable state as the e-channel which are ATM and POS.
An application of HMM requires the specification of two model parameters.

$$(N, M) \qquad (10)$$

and of the three-probability measure (A, B, $\pi$). For convenience, we use the compact notation to indicate the complete parameter for HMMs i.e.

$$\lambda = (A, B, \pi) \qquad (11)$$

where
N is the number of hidden states in HMM model. The state set is denoted by:

$$S_0^T = \{s_1, s_2\} \tag{12}$$

Where N = 2  $S_1$ = High (H) and $S_2$ = Low (L)
M is the number of observations. The set of possible observations is denoted by

$$r_0^T = \{r_1, r_2\} \tag{13}$$

Where $r_1 = ATM$ , and $r_2 = POS$

We define *X* as a fixed state of a sequence of length *T*, and its corresponding observation sequence as: $r_0^T$

$$X = \{X_1, X_2, ..., X_T\} \qquad r_0^T = \{r_1, r_2, ..., r_T\} \tag{14}$$

A is the transition probability matrix of dimension N x N. It stores the probability of state j following the state in the $a_{ij}$ cell:

$$a_{ij} = P(X_{t+1} = s_j \mid r_t = s_i) \tag{15}$$

$$\begin{array}{cc} & X_{t+1}: \; H \quad L \\ X_t: & \begin{array}{c} H \\ L \end{array} \begin{pmatrix} p_{11} & p_{12} \\ p_{21} & p_{22} \end{pmatrix} \end{array}$$

Where H is High and L is Low.
we define $b_i(k)$ as the probability of observation *k* being produced at state *i*, which is independent of time instant *t*.

$$b_i(k) = P(r_k = X_t | X_{t+1} = s_j) \tag{16}$$

$$\begin{array}{cc} & X_{t+1}: ATM \quad POS \\ X_t: & \begin{array}{c} H \\ L \end{array} \begin{pmatrix} p_{11} & p_{12} \\ p_{21} & p_{22} \end{pmatrix} \end{array}$$

$\pi$ is the initial state probability vector.

$$\pi = (\pi_0, \pi_1) \tag{17}$$

$$\pi_0 = p(X_0 = s_i) \tag{18}$$

### 3.2 Estimation Analysis

In estimating the parameters of the hidden Markov model, there are three basic problems:

(i) The Evaluation problem
(ii) The Decoding problem
(iii) The Learning problem

This research work focuses on the decoding problem using the filtering algorithm instead of the popularized Viterbi algorithm.

### 3.2.1 Filtering Method

The filtering method in a hidden Markov model is used to estimate the posterior probabilities of the hidden states given a sequence of observations. The parameters of the filtering method are the transition, emission and the Initial probability matrix.
By combining these parameters, the filtering method is used to estimate the most likely hidden state sequence given a sequence of observations. The parameters of the HMM can be estimated from data using maximum likelihood estimation or other estimation techniques.
The filtering algorithm code for the hidden Markov model is given by:

- Init $P(x_1)$
- Observation update for time 0:

$$P(x_1 \mid r_1) \propto P(r_1 \mid x_1) P(x_1) \tag{19}$$

For t = 1, 2, …

- Time update:

$$P(x_{t+1} \mid r_{1:t}) = \sum P(x_{t+1} \mid x_t) P(x_t \mid r_{1:t}) \tag{20}$$

- Observation update:

$$P(x_{t+1} \mid r_{1:t+1}) \propto P(r_{t+1} \mid x_{t+1}) P(x_{t+1} \mid r_{1:t+1}) \tag{21}$$

#### 3.2.1.1 Filtering Problem

For the filtering problem, we compute $\alpha(X_t) := P(X_t, r_{1:t})$ .

This gives the un-normalized filtered posterior distribution.

Thus, we normalize it by computing $P(X_t \mid r_{1:t}) \propto \alpha(X_t)$.

Starting with $\alpha(X_1) := P(r_1 \mid X_1)P(X_1)$. Note that

$$\begin{aligned}
\alpha(X_t) &= \sum_{X_{t-1}} P(X_t, X_{t-1}, r_{1:t-1}, r_t) \\
&= \sum_{X_{t-1}} P(r_t \mid X_t, X_{t-1}, r_{1:t-1}) P(X_t \mid X_{t-1}, r_{1:t-1}) P(X_{t-1} \mid r_{1:t-1}) \\
&= \sum\nolimits_{X_{t-1}} P(r_t \mid X_t) P(X_t \mid X_{t-1}) P(X_{t-1}, r_{1:t-1}) \\
&= \underbrace{P(r_t \mid X_t)}_{corrector} \underbrace{\sum_{X_{t-1}} P(X_t \mid X_{t-1}) \alpha(X_{t-1})}_{predictor}
\end{aligned} \tag{22}$$

The above is used to solve the filtering problem.

## 4. Results and Discussion

### 4.1 Data

The data to be used for this study is the e-payment transactions data gotten from the website of Central bank of Nigeria. The variables considered in this work are the two e-channel systems which are POS and ATM. The R programming language was used for the statistical analysis. The descriptive statistics of the data used for this study involves the table of statistical measure and the time series plot for the transactions considered; estimating the parameters of the hidden Markov model which involve their transition probabilities, the emission probabilities, initial state probabilities, number of hidden states, number of observable states and their standard errors ; predicting the future state probabilities for the volume of both ATM and POS.

### 4.2 Descriptive Statistics

**Table 1: Descriptive Statistics of Volume of Transactions for ATM and PoS E-Channels from 2009 to 2017**

| **Summary Statistics** | **ATM** | **POS** |
|---|---|---|
| Mean | 30863675 | 2298781 |
| Std. Deviation | 16866748 | 3116571 |
| Skewness | 0.1763947 | 1.518591 |
| Excess Kurtosis | -0.6021409 | 1.175268 |
| Minimum | 2269974 | 1956 |
| Maximum | 66731587 | 11220631 |

Table 1 shows the descriptive statistics of the two e-payment channels: automated teller machine '(ATM) and point of sale (PoS) machine. The mean volume of transaction is 30863675 for ATM and 2298781 for PoS. The minimum volume of transactions for ATM during the period is 2269974 and 1956 for PoS; while the maximum volume of transaction during the period for ATM is 66731587 and 11220631 for PoS. The time plot for both e-payment channels showed an upward trend which is suggestive of the fact that there is a possibility of a surge in the volume of usage in the future. The time plot for both transaction during the specified period is seen in figure 1 below.

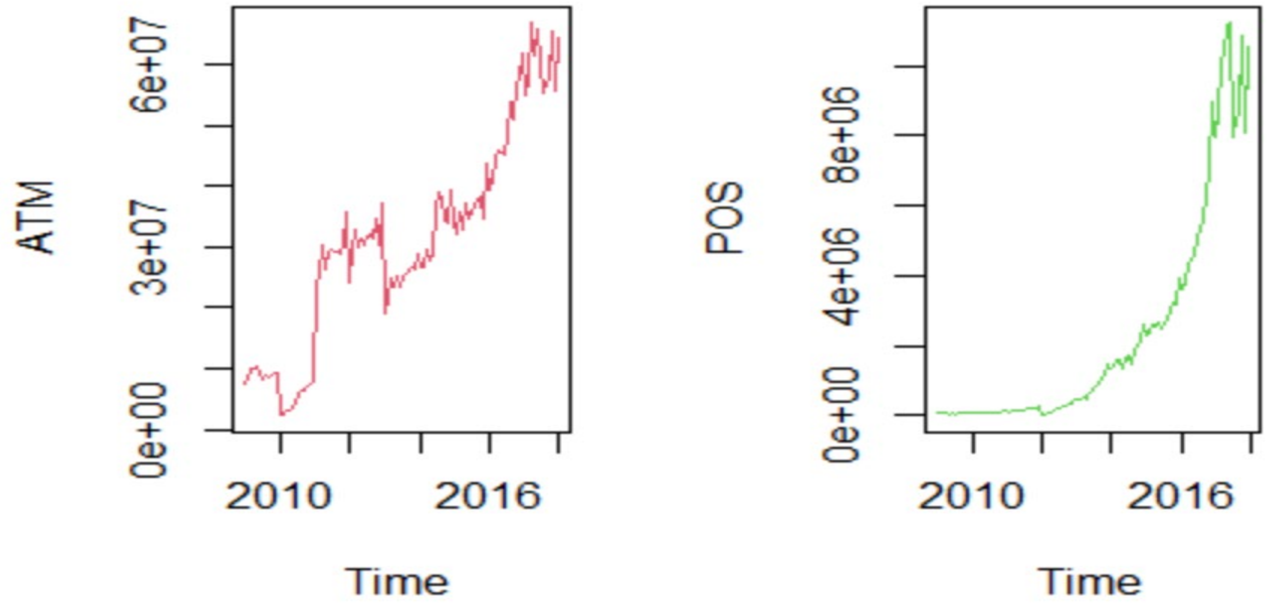


**Figure 1: Time Plot of Volume of Transactions of ATM and PoS from 2009 to 2017**

### 4.3 Parameter Estimates of the HMM for ATM and POS From 2009 to 2017

In estimating the parameters of the hidden Markov model, five parameters were considered: the transition matrix $A$, the emission matrix $B$, the initial state probability $\pi$, the number of hidden states $N$, and number of observable states $M$. The number of hidden states and the number of observable states, have been determined in chapter three. The hidden state is given by $N = \{High, Low\}$ and the observable state is given by $M = \{ATM, POS\}$.

The result for the initial state probability is given by:

$$\pi = (\pi_0, \pi_1) = \{1,0\} \tag{23}$$

Fitting the hidden Markov model, produced the transition probabilities given by:

$$\begin{array}{cc} & \overbrace{\begin{array}{cc} X_{t+1}:L & H \end{array}} \\ X_t: \begin{array}{c} L \\ H \end{array} & \begin{pmatrix} 0.944 & 0.056 \\ 0.008 & 0.992 \end{pmatrix} \end{array} \tag{24}$$

After finding an HMM model for 2 hidden states, we get the above transition matrix.

From the transition probability, it is seen that the probability of the volume of transactions for both ATM and POS to remain low is 0.944 (94.4%) and the probability of transiting from low to high volume is 0.056 (5.6%). Also, the probability of transition from high to low volume of transaction is 0.008 (0.8%) and the probability of having only high transactions is 0.992 (99.2%). This means that the probability of remaining in the same state is higher than the probability of transiting from one state to another state. In addition, the emission matrix for the data used for this work is given by:

$$\begin{array}{cc} & \overbrace{\begin{array}{cc} X_{t+1}:ATM & POS \end{array}} \\ X_t: \begin{array}{c} L \\ H \end{array} & \begin{pmatrix} 0.5053 & 0.4947 \\ 0.4292 & 0.5708 \end{pmatrix} \end{array} \tag{25}$$

From the emission matrix, the probability that the volume of the transaction would be low given that it is an ATM transaction is 0.5053; the probability for a low transaction given that it is a POS transaction is 0.4947. The probability that the volume of the transaction would be high given that it is an ATM transaction is 0.4292, and the probability that the volume of the transaction would be high given that it is a POS transaction is 0.5708.

The transition diagram for the fit matrix is seen in figure 2 below.

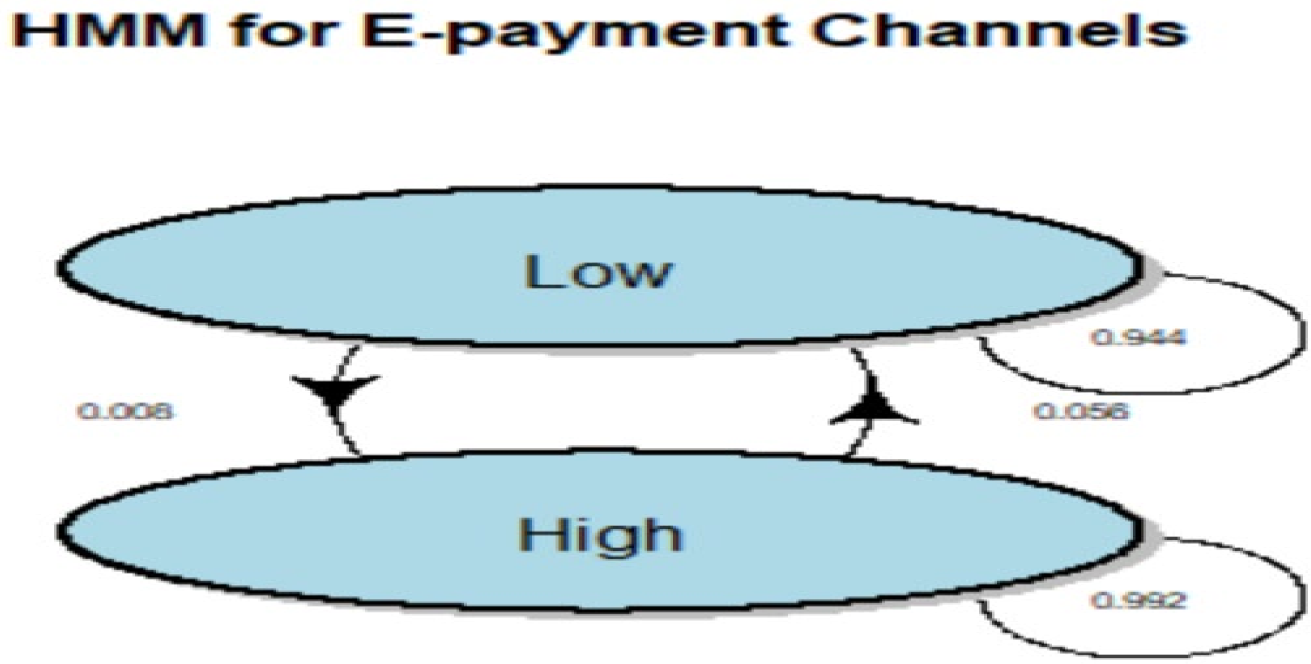


**Figure 2: Transition Diagram for ATM and PoS Volume Transactions from 2009 to 2017**

### 4.4 Prediction of Atm and Pos Transaction Volume with the Model

The prediction of the future trend of the use of ATMs and PoS revealed that PoS transactions would be more popularized compared to the ATM transaction channel. The posterior probability of the prediction is seen in the table 2.

**Table 2: Posterior Prediction of Volume of Transactions for ATM and PoS E-Channels**

| Time (t) | ATM | POS |
|---|---|---|
| 1 | $1.00\times10^{0}$ | 0.00 |
| 2 | $9.99\times10^{-1}$ | $2.07\times10^{-7}$ |
| 3 | $9.99\times10^{-1}$ | $3.57\times10^{-6}$ |
| 4 | $9.99\times10^{-1}$ | $2.80\times10^{-6}$ |
| 5 | $9.99\times10^{-1}$ | $5.11\times10^{-6}$ |
| . | … | … |
| . | … | … |
| . | … | … |
| 104. | $2.69\times10^{-20}$ | $1.00\times10^{0}$ |
| 105. | $8.81\times10^{-21}$ | $1.00\times10^{0}$ |
| 106. | $6.81\times10^{-44}$ | $1.00\times10^{0}$ |
| 107. | $3.35\times10^{-17}$ | $1.00\times10^{0}$ |
| 108. | $1.96\times10^{-40}$ | $1.00\times10^{0}$ |

### 4.5 Steady State

A steady-state behaviour of a Markov chain is the long-term probability that the system will be in each state. In the matrix below, it is seen that in the future, the probability of a higher volume of e-payment transactions using POS is higher than that of ATM. This is consistent with the prediction made in (25) above. The steady state probability is seen in (26) below.

$$\pi = \begin{pmatrix} ATM & POS \\ 0.125 & 0.875 \end{pmatrix} \tag{26}$$

### 4.6 Model Performance

The standard error of the estimate was used to evaluate the model performance. The standard error values are shown below.

ATM PoS

$$S.E = \begin{pmatrix} 4.33\times10^{-3} & 1.51\times10^{-3} \\ 1.51\times10^{-3} & 4.33\times10^{-3} \end{pmatrix} \tag{27}$$

The mean square error is given by:

$$\begin{array}{cc} & \begin{array}{cc} \text{ATM} & \text{PoS} \end{array} \\ MSE = & \begin{pmatrix} 4.11\times10^{-4} & 1.68\times10^{-4} \\ 1.68\times10^{-3} & 4.11\times10^{-4} \end{pmatrix} \end{array} \tag{28}$$

This implies that the estimates for the transition probability for the model are better estimates since the values of the standard errors are close to zero to two decimal places. The closer the values are to zero, the better the estimates of the model. This is also true for the mean square errors (MSE).

## 5. Conclusion

In this work, an HMM is proposed in e-payment transactions to see the trends of these e-payment systems and to determine what would be the future trend of the e-payment channels. Two e-payment channels were considered: Automated teller machine (ATM) and point of sale (PoS) machines which are the observable state while the hidden states are high and a low volume of transactions. The emission matrix was determined based on the observable state count and the transition probability count. The filtering method was employed in the HMM to estimate the parameters of the model. From the empirical analysis, it is predicted that PoS transactions would overtake that of the ATM in the future. The model performance in addition showed that the hidden Markov model is a parsimonious model as the standard errors and the mean square errors are all close to zero correct to two decimal places based on the data used in this study.

The steady state probability also supported this claim as the probability that Nigerians would use PoS is 0.875 (87.5%) and the probability that Nigerians would use ATM is 0.125 (12.5%) based on the data used in this study.

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